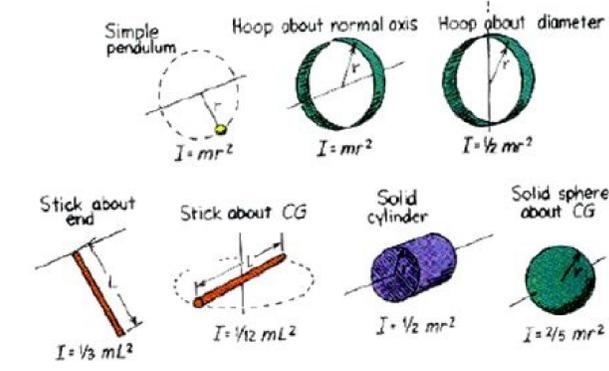
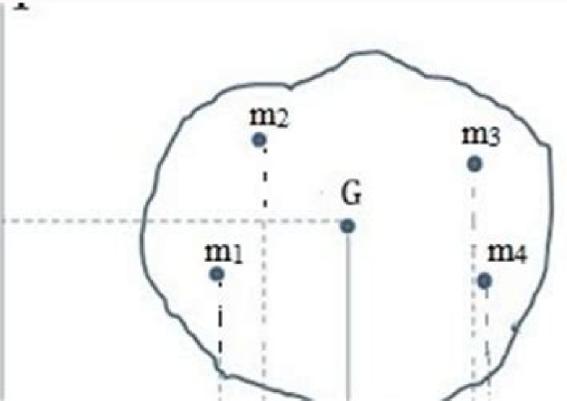
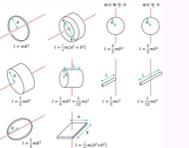


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$$I \triangleq \sum_{i=1}^N m_i r_i^2$$



**Objective:**

"To determine the mass moment of inertia of flywheel"

**Apparatus:**

Flywheel, Thread, Weight, Hanger, Stop Watch and Meter rod.

**Theory:**

**Flywheel:**

A flywheel is a mechanical device with a significant moment of inertia used as a storage device for rotational energy. Flywheels resist changes in their rotational speed, which helps steady the rotation of the shaft when a fluctuating torque is exerted on it by its power source such as that caused by a reciprocating engine.



**Mass:**

Mass is the quantity of matter that is present in a body. Mass of a body is the measure of its inertia.

**Inertia:**

Inertia is that property of a body due to which it resists change in its state of rest or of uniform motion.

Mass moment of inertia.

**Learning Objectives** Calculate the moment of inertia for uniformly shaped, rigid bodies Apply the parallel axis theorem to find the moment of inertia about any axis parallel to one already known Calculate the moment of inertia for compound objects In the preceding subsection, we defined the moment of inertia but did not show how to calculate it. In this subsection, we show how to calculate the moment of inertia for several standard types of objects, as well as how to use known moments of inertia to find the moment of inertia for a shifted axis or for a compound object. This section is very useful for seeing how to apply a general equation to complex objects (a skill that is critical for more advanced physics and engineering courses). We defined the moment of inertia  $I$  of an object to be  $I = \sum m_i r_i^2$  for all the point masses that make up the object. Because  $r$  is the distance to the axis of rotation from each piece of mass that makes up the object, the moment of inertia for any object depends on the chosen axis. To see this, let's take a simple example of two masses at the end of a massless (negligibly small mass) rod (Figure 10.19) and calculate the moment of inertia about two different axes. In this case, the summation over the masses is simple because the two masses at the end of the barbell can be approximated as point masses, and the sum therefore has only two terms. In the case with the axis in the center of the barbell, each of the two masses  $m$  is a distance  $R$  away from the axis, giving a moment of inertia of  $I = mR^2 + mR^2 = 2mR^2$ . In the case with the axis at the end of the barbell—passing through one of the masses—the moment of inertia is  $I = m(0)^2 + m(2R)^2 = 4mR^2$ . From this result, we can conclude that it is twice as hard to rotate the barbell about the end than about its center. Figure 10.19(a) shows a barbell with an axis of rotation through its center; (b) shows a barbell with an axis of rotation through one end. In this example, we had two point masses and the sum was simple to calculate. However, to deal with objects that are not point-like, we need to think carefully about each of the terms in the equation. The equation asks us to sum over each "piece of mass" a certain distance from the axis of rotation. But what exactly does each "piece of mass" mean? Recall that in our derivation of this equation, each piece of mass had the same magnitude of velocity, which means the whole piece had to have a single distance  $r$  to the axis of rotation. However, this is not possible unless we take an infinitesimally small piece of mass  $dm$ , as shown in Figure 10.20. Using an infinitesimally small piece of mass to calculate the contribution to the total moment of inertia. The need to use an infinitesimally small piece of mass  $dm$  suggests that we can write the moment of inertia by evaluating an integral over infinitesimal masses rather than doing a discrete sum over finite masses:  $I = \sum m_i r_i^2$  becomes  $I = \int r^2 dm$ . This, in fact, is the form we need to generalize the equation for complex shapes. It is best to work out specific examples in detail to get a feel for how to calculate the moment of inertia for specific shapes. This is the focus of most of the rest of this section. Consider a uniform (density and shape) thin rod of mass  $M$  and length  $L$ , as shown in Figure 10.21. We want a thin rod so that we can assume the cross-sectional area of the rod is small and the rod can be thought of as a string of masses along a one-dimensional straight line. In this example, the axis of rotation is perpendicular to the rod and passes through the midpoint for simplicity. Our task is to calculate the moment of inertia about this axis. We orient the axes so that the  $x$ -axis is the axis of rotation and the  $x$ -axis passes through the length of the rod, as shown in the figure. This is a convenient choice because we can then integrate along the  $x$ -axis. Figure 10.21 shows the rod and the axis. We define  $dm$  to be a small element of mass making up the rod. The moment of inertia integral is an integral over the mass distribution. However, we know how to integrate over space, not over mass. We therefore need to find a way to relate mass to spatial variables. We do this using the linear mass density  $\lambda$  of the object, which is the mass per unit length. Since the mass density of this object is uniform, we can write  $\lambda = \frac{M}{L}$ ; or  $m = \lambda l$ . If we take the differential of each side of this equation, we find  $dm = \lambda dl$  since  $\lambda$  is constant. We chose to orient the rod along the  $x$ -axis for convenience—this is where that choice becomes very helpful. Note that a piece of the rod  $dl$  lies completely along the  $x$ -axis and has a length  $dx$ ; in fact,  $dl = dx$  in this situation. We can therefore write  $dm = \lambda dx$ , giving us an integration variable that we know how to deal with. The distance of each piece of mass  $dm$  from the axis is given by the variable  $x$ , as shown in the figure. Putting this all together, we obtain  $I = \int r^2 dm = \int x^2 \lambda dx = \lambda \int x^2 dx$ . The last step is to be careful about our limits of integration. The rod extends from  $x = -\frac{L}{2}$  to  $x = \frac{L}{2}$ , since the axis is in the middle of the rod at  $x = 0$ . This gives us  $I = \lambda \int_{-L/2}^{L/2} x^2 dx = \lambda \left[ \frac{x^3}{3} \right]_{-L/2}^{L/2} = \lambda \left( \frac{L^3}{24} - \left( -\frac{L^3}{24} \right) \right) = \lambda \frac{L^3}{12} = \frac{M}{L} \frac{L^3}{12} = \frac{1}{12} ML^2$ . Next, we calculate the moment of inertia for the same uniform thin rod but with a different axis choice so we can compare the results. We would expect the moment of inertia to be smaller about an axis through the center of mass than the endpoint axis, just as it was for the barbell example at the start of this section. This happens because more mass is distributed farther from the axis of rotation. Now consider the same uniform thin rod of mass  $M$  and length  $L$ , but this time we move the axis of rotation to the end of the rod. We wish to find the moment of inertia about this new axis (Figure 10.22). Calculation of the moment of inertia  $I$  for a uniform thin rod about an axis through the end of the rod. The quantity  $(dm)$  is again defined to be a small element of mass making up the rod. Just as before, we obtain  $I = \int r^2 dm = \int x^2 \lambda dx = \lambda \int_0^L x^2 dx = \lambda \left[ \frac{x^3}{3} \right]_0^L = \lambda \frac{L^3}{3} = \frac{M}{L} \frac{L^3}{3} = \frac{1}{3} ML^2$ . Note the rotational inertia of the rod about its endpoint is larger than the rotational inertia about its center (consistent with the barbell example) by a factor of four. The similarity between the process of finding the moment of inertia of a rod about an axis through its middle and about an axis through its end is striking, and suggests that there might be a simpler method for determining the moment of inertia for a rod about any axis parallel to the axis through the center of mass. Such an axis is called a parallel axis. There is a theorem for this, called the parallel-axis theorem, which we state here but do not derive in this text. **Parallel-Axis Theorem** Let  $m$  be the mass of an object and let  $d$  be the distance from an axis through the object's center of mass to a new axis. Then we have  $I = I_{cm} + md^2$ . Let's apply this to the uniform thin rod with axis example solved above.  $I_{cm} = \frac{1}{12} ML^2$  and  $I_{end} = \frac{1}{3} ML^2$ . The parallel-axis theorem gives  $I_{end} = I_{cm} + md^2 = \frac{1}{12} ML^2 + M \left( \frac{L}{2} \right)^2 = \frac{1}{12} ML^2 + \frac{1}{4} ML^2 = \frac{1}{3} ML^2$ . This result agrees with our more lengthy calculation (Equation 10.21). Solution For the child,  $I = \frac{1}{2} m r^2$ , and for the merry-go-round,  $I = \frac{1}{2} m r^2$ . Therefore  $I_{total} = 25(1)^2 + \frac{1}{2}(500)(2)^2 = 500(2)^2 = 25 + 1000 = 1025$  kg $\cdot$ m $^2$ . Significance The value should be close to the moment of inertia of the merry-go-round by itself because it has much more mass distributed away from the axis than the child does. Example 10.21: Rod and Solid Sphere Find the moment of inertia of the rod and solid sphere combination about the two axes as shown below. The rod has length 0.5 m and mass 2.0 kg. The radius of the sphere is 20.0 cm and has mass 1.0 kg. Strategy Since we have a compound object in both cases, we can use the parallel-axis theorem to find the moment of inertia about each axis. In (a), the center of mass of the sphere is located at a distance  $(L + R)$  from the axis of rotation. In (b), the center of mass of the sphere is located a distance  $R$  from the axis of rotation. In both cases, the moment of inertia of the rod is about an axis at one end. Refer to Table 10.4 for the moments of inertia for the individual objects.  $I_{total} = I_{rod} + I_{sphere} = I_{rod} + I_{cm,sphere} + m_{sphere} d^2 = I_{rod} + \frac{1}{2} m_{sphere} R^2 + m_{sphere} (L + R)^2 = \frac{1}{2} (2.0 \text{ kg}) (0.5 \text{ m})^2 + (1.0 \text{ kg}) (0.5 \text{ m} + 0.2 \text{ m})^2 + \frac{1}{2} (1.0 \text{ kg}) (0.2 \text{ m})^2 + (1.0 \text{ kg}) (0.2 \text{ m})^2 = 0.25 \text{ kg}\cdot\text{m}^2 + 0.673 \text{ kg}\cdot\text{m}^2 + 0.02 \text{ kg}\cdot\text{m}^2 + 0.02 \text{ kg}\cdot\text{m}^2 = 1.0 \text{ kg}\cdot\text{m}^2$ . (b)  $I_{total} = I_{rod} + I_{cm,sphere} + m_{sphere} d^2 = \frac{1}{12} (2.0 \text{ kg}) (0.5 \text{ m})^2 + \frac{1}{2} (1.0 \text{ kg}) (0.2 \text{ m})^2 + (1.0 \text{ kg}) (0.2 \text{ m})^2 = 0.042 \text{ kg}\cdot\text{m}^2 + 0.02 \text{ kg}\cdot\text{m}^2 + 0.02 \text{ kg}\cdot\text{m}^2 = 0.082 \text{ kg}\cdot\text{m}^2$ . Significance Using the parallel-axis theorem eases the computation of the moment of inertia of compound objects. We see that the moment of inertia is greater in (a) than (b). This is because the axis of rotation is closer to the center of mass of the system in (b). The simple analogy is that of a rod. The moment of inertia about one end is  $\frac{1}{3} mL^2$ , but the moment of inertia through the center of mass along its length is  $\frac{1}{12} mL^2$ . Example 10.22: Angular Velocity of a Pendulum A pendulum in the shape of a rod (Figure 10.23) is released from rest at an angle of  $30^\circ$ . It has a length 30 cm and mass 300 g. What is its angular velocity at its lowest point? Figure 10.23: A pendulum in the form of a rod is released from rest at an angle of  $30^\circ$ . Strategy Use conservation of energy to solve the problem. At the point of release, the pendulum has gravitational potential energy, which is determined from the height of the center of mass above its lowest point in the swing. At the bottom of the swing, all of the gravitational potential energy is converted into rotational kinetic energy. Solution The change in potential energy is equal to the change in rotational kinetic energy.  $\Delta U = \Delta K = 0$ . At the top of the swing:  $U = mgh$ ,  $h = mgL^2 (\cos \theta)$ . At the bottom of the swing,  $U = mgL^2$ . At the top of the swing, the rotational kinetic energy is  $K = 0$ . At the bottom of the swing,  $K = \frac{1}{2} I \omega^2$ . Therefore,  $\Delta U + \Delta K = 0 \Rightarrow mgL^2 (1 - \cos \theta) = \frac{1}{2} I \omega^2$ . Solving for  $\omega$ , we have  $\omega = \sqrt{\frac{2mgL(1 - \cos \theta)}{I}}$ . Inserting numerical values, we have  $\omega = \sqrt{\frac{2(9.8 \text{ m/s}^2)(0.3 \text{ m})(1 - \cos 30^\circ)}{\frac{1}{12}(0.3 \text{ kg})(0.3 \text{ m})^2}} = 3.6 \text{ rad/s}$ . Significance Note that the angular velocity of the pendulum does not depend on its mass.

